

Use of spatial cross correlation function to study formation mechanism of massive elliptical galaxies

Tuli De¹, Tanuka Chattopadhyay²

and

Asis Kumar Chattopadhyay³

¹Department of Mathematics, Heritage Institute of Technology, Kolkata
Choubaga Road, Anandapur, Kolkata -700107, India
email: tuli.de@heritageit.edu

²Department of Applied Mathematics, Calcutta University, Kolkata, India
92 A.P.C. Road, Kolkata -700009
email: tanuka@iucaa.ernet.in

³Department of Statistics, Calcutta University, Kolkata
35 Ballygunge Circular Road, Kolkata-700019, India
email: akcstat@caluniv.ac.in

Abstract

Spatial clustering nature of galaxies have been studied previously through auto correlation function. The same type of cross correlation function has been used to investigate parametric clustering nature of galaxies e.g. with respect to masses and sizes of galaxies.

Here formation and evolution of several components of nearby massive early type galaxies ($M_* \geq 1.3 \times 10^{11} M_\odot$) have been envisaged through cross correlations, in the mass-size parametric plane, with high redshift ($0.2 \leq z \leq 7$) early type galaxies (hereafter ETG). It is found that the inner most components of nearby ETGs have significant correlation ($\sim 0.5 \pm (0.02 - 0.07)$) with ETGs in the highest redshift range ($2 \leq z \leq 7$) called ‘red nuggets’ whereas intermediate components are highly correlated ($\sim 0.65 \pm (0.03 - 0.07)$) with ETGs in the redshift range $0.5 \leq z \leq 0.75$. The outer most part has no correlation in any range, suggesting a scenario through in situ accretion.

The above formation scenario is consistent with the previous results obtained for NGC5128 (Chattopadhyay et al. (2009); Chattopadhyay et al. (2013)) and to some extent for nearby elliptical galaxies (Huang et al. (2013)) after considering a sample of ETGs at high redshift with stellar masses greater than or equal to $10^{8.73} M_\odot$. So the present work indicates a three phase formation of massive nearby elliptical galaxies instead of two as discussed in previous works.

Keywords:: Cross correlation, Elliptical galaxies, Clustering.

1. Introduction

In 1934 Hubble observed that the frequency distribution of the count of galaxies over the space is strongly skew but the distribution of its logarithm is close to symmetric. Bok (1934) and Mowbray (1938) found that variance of the count is considerably larger than expected for a random galaxy distribution. Such studies indicates that locally galaxies are clustered

over space. Several attempts have been made to study this clustering nature on the basis of angular positions of the galaxies. Most of them (Zwicky(1953), Limber (1953,1954), Chandrasekhar and Munch (1952)) have used spatial and angular correlation functions to study this phenomenon. In this area the contributions of Neyman and Scott(1954) is very significant. This spatial clustering nature motivated us to investigate the clustering nature with respect to the other parameters also by using the same approach.

Classical formation of elliptical galaxies can be divided into five major categories e.g. (i) the monolithic collapse model, (Larson (1975); Carberg (1984); Arimoto and Yoshii (1987)) (ii) the major merger model (Toomre (1977); Ashman and Zepf (1992); Zepf et al. (2000); Bernardi et al. (2011); Prieto et al. (2013)), (iii) the multiphase dissipational collapse model (Forbes (1997)), (iv) the dissipationless merger model (Bluck et al. (2012); Newman et al. (2012)) and (v) the accretion and in situ hierarchical merging (Mondal et al. (2008)). Recent observations in the deep field have explored that high redshift galaxies have the size of the order of 1 kpc (Daddi et al. (2005); Trujillo et al. (2006); Damjanov et al. (2011)) and have higher velocity dispersion (Cappellari et al. (2009); Onodera et al. (2009)) than nearby ETGs of the same stellar mass. Galaxies at intermediate redshifts (since $z \approx 2.5$) have stellar masses and sizes increased by a factor almost 3-4 (Van Dokkum et al. (2010); Papovich et al. (2012); Szomoru et al. (2012)). All these evidences suggest that massive ETGs form in two phases viz. inside-out i.e. intense dissipational process like accretion (Dekel et al. (2009)) or major merger form an initially compact inner part. After this a second slower phase starts when the outer most part is developed through non-dissipational process e.g. dry, minor merger. The above development arising both in the field of observations as well as theory, severely challenge classical models like monolithic collapse or major merger and favors instead a “two phase” scenario (Oser et al. (2010); Johanson et al. (2012)) of the formation of nearby elliptical galaxies. The task remains, is to check whether the compact inner parts of the nearby ETGs have any kind of similarity with the fossil bodies (viz. ‘red nuggets’) at high redshift.

In a previous work (Huang et al. (2013)), the authors have pursued the above task through matching ‘median’ values of the two systems. They used this measure with respect to univariate data and the univariate data they considered, are either stellar ‘mass’ or ‘size’. For ETGs in the redshift range, $0.5 \leq z \leq 7$, considered, in the present data set, the stellar mass-size correlation is $r(M_*, R_e) = 0.391$, p-value=0.00. For nearby ETGs for inner, intermediate and outer components the stellar mass-size correlations with p-values are $r(M_*, R_e) = 0.720$, p-value=0.00, $r(M_*, R_e) = 0.636$, p-value=0.00, $r(M_*, R_e) = 0.573$, p-value=0.00 respectively and all these values are highly significant. Hence use of univariate median matching is not sufficient in the present context for highly correlated bivariate data. Also, median does not include all objects in a particular data set. For this a more sophisticated technique is in demand for such kind of investigation.

In the present work we have used the mass-size data of high red shift galaxies and nearby ETGs and used a cross-correlation, especially designed to study bivariate data. This is more trustworthy and meaningful in the present situation. In section 2 we have discussed the data set and in section 3 we have described the method. The results and interpretations are given under section 4.

2. Data sets:

we have considered eight data sets. Data sets 1-3 consist of stellar masses and sizes of 70 nearby ETGs taken from Ho et al. (2011). There are three components corresponding to each massive ETG, described by a single Sérsic (1968) index, as considered by Huang et al. (2013). They are, (i) an inner component with effective radii $R_e \leq 1$ kpc, (ii) an intermediate component with effective radii $R_e \sim 2.5$ kpc and (iii) an outer envelope with $R_e \sim 10$ kpc. Data sets 4-8 consist of stellar masses and effective radii of high redshift ETGs with stellar masses $M_* \geq 10^{8.73} M_\odot$ in the redshift bins $0.5 < z \leq 0.75$, $0.75 < z \leq 1$, $1 < z \leq 1.4$, $1.4 < z \leq 2.0$, $2.0 < z \leq 2.7$. Unlike Huang et al. (2013) we also included intermediate mass high redshift galaxies. Data sets 4-8 contains 786 high redshift ETGs from the following works. 392 galaxies ($0.2 \leq z \leq 2.7$) from Damjanov et al. (2011), 32 ($1.5 < z < 3$) galaxies from GOODS-NICMOS survey (Conselice et al. (2011)) for Sérsic (1968) index $n > 2$, 21 galaxies from CANDELS (Grogin et al. 2011) ($1.5 < z < 2.5$), 107 from Papovich et al. (2012) ($1.5 \leq z \leq 2.5$), 48 from McLure et al. (2012) ($1.3 < z < 1.5$), 62 from Saracco et al. (2011) ($1 < z < 2$), 124 galaxies from Nilsson et al. (2013).

Since the data sets are chosen from different sources, they have various selection biases and errors etc. Hence, to judge their compatibility we have performed a multivariate multi sample matching test (Puri & Sen (1966): Appendix, McKean (1974)) to see whether they have the same parent distribution or not. From previous works it is evident that galaxies have undergone cosmological evolution via merger or accretion (Naab (2013); Khochfar & Silk (2006); De Lucia & Blaizot (2007); Guo & White (2008); Kormendy et al. (2009); Hopkins et al. (2010)) and we have performed the matching test for galaxies within the same redshift zone. The data set taken from Damjanov et al. (2011) contains maximum number of galaxies within the entire redshift zone ($0.2 \leq z \leq 2.7$) used in the present analysis. For this we have compared it with the other sets. The results are given in Table 1. It is clear from Table 1 that all the tests are accepted except one (Papovich et al. (2012)) where the matching redshift zone is very narrow. Since almost in 99% cases the test is accepted we assume that the dataset consisting of samples from different sources is more or less homogeneous with respect to mass-size plane.

It is to be noted that in Ho et al. (2011) paper, the magnitude values of the three components corresponding to each ETG are given from which, luminosities are computed. Then these luminosities are multiplied by (M/L) ratios for obtaining stellar masses. The (M/L) ratios are computed following Bell et al. (2001). we have not become able to retrieve data for some high redshift galaxies and instead included some new data from other recent references so that sample size of high redshift galaxies are somewhat reduced in our case, but the overall distribution of these galaxies are similar in the size-redshift plane with those considered by Huang et al. (2013a) (viz. Fig.1 of Huang et al. (2013a) and Fig.1 in the present work) except the region $1 \leq z \leq 2$ which is more populated than Huang et al. (2013a) sample as we have included new galaxies in data sets 4-8.

3. Method:

The theory of the spatial distribution of galaxies has been discussed by several authors like Peebles (1980), Blake et al. (2006), Martinez and Saar (2012) etc. During 1950s, the most extensive statistical study was performed by Neyman and Scott. Their work was based on the large amount of data obtained from the LICK survey. The main empirical statistics they

used were the angular auto correlation function of the galaxy counts(Neyman et al. (1956)) and Zwicky's index of clumpiness (Neyman et al. (1954)).

Neyman and Scott (1952) introduced this theory on the basis of four assumptions viz. (i) galaxies occur only in clusters, (ii) The number of galaxies varies from cluster to cluster subject to a probabilistic law, (iii) the distribution of galaxies within a cluster is also subject to a probabilistic law and iv) the distribution of cluster centres in space is subject to a probabilistic law described as quasi-uniform. As the observed distribution of number of galaxies does not follow Poisson law, it is suspected that not only the apparent but also the actual spatial distribution of galaxies is clustered.

In the present work attempts have been made to establish the same postulates with respect to mass-size distribution of galaxies. Here the hypothesis is "there is clustering nature also in the galaxy distribution with respect to the parameters mass and size of the galaxies". This particular hypothesis also has been studied by several authors. But we have followed the same approach as that used to establish spatial clustering as discussed above.

In cosmology the cross correlation function $\xi(r)$ of a homogeneous point process is defined by (Peebles (1980))

$$dP_{12} = \pi^2[1 + \xi(r)]dV_1dV_2 \quad (3.1)$$

where r is the separation vector between the points x_1 and x_2 and π is mean number density. Considering two infinitesimally small spheres centered in x_1 and x_2 with volumes dV_1 and dV_2 , the joint probability that in each of the spheres lies a point of the point process is

$$dP_{12} = \lambda_2(x_1, x_2)dV_1dV_2 \quad (3.2)$$

In (2), $\lambda_2(x_1, x_2)$ is defined as the second order intensity function of a Point process.

If the point field is homogeneous, the second-order intensity function $\lambda_2(x_1, x_2)$ depends only on the distance $r = |x_1 - x_2|$ and direction of the line passing through x_1 and x_2 . If, in addition, the process is isotropic, the direction is not relevant and the function only depends on r and may be denoted by $\lambda_2(r)$. Then

$$\xi(r) = \frac{\lambda_2(r)}{\pi^2} - 1 \quad (3.3)$$

Different authors proposed several estimators of $\xi(r)$. Natural estimators have been proposed by Peebles and Hauser (1974). The cross correlation function $\xi(r)$ can be estimated from the galaxy distribution by constructing pair counts from the data sets. A pair count between two galaxy populations 1 and 2, $D_1D_2(r)$, is a frequency corresponding to separation r to $r+\delta r$ for a bin of width δr in the histogram of the distribution r , D_iR_j and R_iR_j denote the same pair count corresponding to one galaxy sample and one simulated sample and two simulated samples respectively, $i, j=1, 2$. Two natural estimators are given by

$$\hat{\xi}_1 = \frac{D_1D_2(r)}{D_2R_1(r)} - 1 \quad (3.4)$$

$$\hat{\xi}_2 = \frac{D_1D_2(r)}{D_1R_2(r)} - 1 \quad (3.5)$$

Another two improved estimators are(Blake et al(2006))

$$\hat{\xi}_3 = \frac{D_1D_2(r)R_1R_2(r)}{D_1R_2(r)D_2R_1(r)} - 1 \quad (3.6)$$

and

$$\hat{\xi}_4 = \frac{D_1 D_2(r) - D_1 R_2(r) - D_2 R_1(r) + R_1 R_2(r)}{R_1 R_2(r)} \quad (3.7)$$

The first two estimates are potentially biased. As in the present situation we are considering mass-size parametric space, we have taken r as the Euclidean distance between two (mass,size) points of two galaxies either original or simulated. In order to generate simulated samples of mass and size, we have used uniform distribution of mass and size with ranges selected from original samples. Here r is normalized by dividing the original separation by the maximum separation. The variances of the estimators are measured by bootstrap method.

4. Results and discussion:

We have computed the cross-correlation functions of each of data sets 1-3 with data sets 4-8 i.e. we have tried to find any kind of correlation between three components of nearby ETGs with high redshift ETGs in five redshift bins as mentioned above. We have found significant correlation between data set 1 and data set 8 and between data set 2 and data set 4. This is clear from Figs. 2 and 3 respectively where the correlations are as high as $0.5 \pm (0.02 - 0.07)$ and $0.65 \pm (0.03 - 0.07)$ for both the estimates at minimum separation (*viz.* $r \sim 0.1$). These show that the innermost components of nearby elliptical galaxies are well in accordance with highest redshift massive ETGs (*viz.* mass $\sim 10^{11.14} M_\odot$ and $R_e \sim 0.92$ kpc), known as ‘red nuggets’ but the intermediate components are highly correlated with galaxies in the redshift bin $0.5 \leq z \leq 0.75$ having median mass and size, $10^{10.87} M_\odot$ and 2.34 kpc respectively. If we merge data sets 1 and 2 and compare with high z galaxies in five redshift bins, the cross-correlation functions are all close to zero at all separations unlike Huang et al (2013a).

The above result is somewhat consistent with the work of Huang et al. (2013a) in a sense that the inner and intermediate parts are the fossil evidences of high red shift galaxies but unlike Huang et al. (2013a), components 1 and 2 together show no correlation with all high redshift ETGs together they are highly correlated with high redshift ETGs in two red shift bins and this indicates clearly two different epochs of structure formation as shown by their z values.

After finding the cross correlation functions between data sets 1 and 8, we have fitted a power law assuming the relation

$$\xi(r) \propto \frac{1}{r} \quad (4.8)$$

i.e.,

$$\xi(r) = A r^{-1} \quad (4.9)$$

Where for estimator 1, $A = 0.02672$ and for estimator 2, $A = 0.031395$.

We have also performed Kolmogorov Smirnov test for justifying the goodness of fit of the power law. Here we have assumed that the cross correlations and the fitted values are samples coming from the distribution function of a Pareto distribution. The p-values for this test for estimator 1 is 0.4175 and for estimator 2 is 0.7869, signifying that the tests are accepted and the fitted power law gives well justification for the cross correlation and distance relationship. We have fitted similar power laws for cross correlation function and distance for data sets 2 and 4. Here the proportionality constants are 0.0247 for estimator 1

and 0.0369 for estimator 2. The Kolmogorov Smirnov tests give p-values = 0.7869 for both the estimators, signifying that in this case also the relationship is well justified.

On the other hand, cross-correlation function, computed for data set 3 with galaxies in the above five bins are all insignificant which is clear from Fig. 4.

During the formation of massive ellipticals, major and minor merger play a significant role for the morphological and structural evolution (Naab (2013); Khochfar & Silk (2006); De Lucia & Blaizot (2007); Guo & White (2008); Kormendy et al. (2009); Hopkins et al. (2010)). The most massive elliptical galaxies (or their progenitors) are considered to start their evolution at $z \sim 6$ or higher in a dissipative environment and rapidly become massive ($\sim 10^{11} M_\odot$) and compact at $z \sim 2$ (Dekel et al. (2009); Oser et al. (2010); Feldmann et al. (2011); Oser et al. (2012)). Also a significant fraction is observed to be quiescent at $z \sim 2$, 4-5 times more compact and a factor of two less massive than their low redshift descendants (van Dokkum et al. (2008); Cimatti et al. (2008); Bezanson et al. (2009); van Dokkum et al. (2010); Whitaker et al. (2012)). Now, for the massive ellipticals in the present sample, the innermost cores (data set 1) are well in accordance with highest redshift ($2.0 < z \leq 2.7$) galaxies and their core masses (viz. median value $\sim 10^{10.203} M_\odot$ and $10^{10.6839} M_\odot$ respectively). Hence it is reasonable to separate that these high redshift population forms the cores of at least some, if not all, present day massive ellipticals. Thus formation of massive ellipticals only by monolithic collapse model is challenged because they will be too small and too red (van Dokkum et al. (2008); Ferré-Mateu et al. (2012)), the subsequent evolution forming the intermediate (data set 2) and outer part (data set 3) might be as follows. On the aspect of major or minor major, following Naab et al. (2009) it is seen that if M_i and r_i be the mass and radius of a compact initial stellar system with a total energy E_i and mean square speed $< v_i^2 >$ and M_a, r_a, E_a and $< v_a^2 >$ be the corresponding values after merger with other systems then,

$$\frac{< v_f^2 >}{< v_i^2 >} = \frac{(1 + \eta_\varepsilon)}{1 + \eta} \quad (4.10)$$

$$\frac{r_{g,f}}{r_{g,i}} = \frac{(1 + \eta)^2}{(1 + \eta_\varepsilon)} \quad (4.11)$$

$$\frac{\rho_f}{\rho_i} = \frac{(1 + \eta_\varepsilon)^3}{(1 + \eta)^5} \quad (4.12)$$

where the quantities with suffix ‘f’ are the final values, $\eta = \frac{M_a}{M_i}$, $\varepsilon = < v_a^2 > / < v_i^2 >$, ρ is the density. Then for $\eta=1$ (major merger), the mean square speed remains same, the size increases by a factor of 2 and densities drop by a factor of four. Now, in the present situation, the intermediate part (data set 2) has radii (median value $< R_e > \sim 2.560$ kpc) which is almost 3 times larger than the radii of inner part (median value $< R_e > \sim 0.6850$ kpc).

Also in a previous work (Chattopadhyay et al. (2009); Chattopadhyay et al. (2013)) on the brightest elliptical galaxy NGC 5128, we have found three groups of globular clusters. One is originated in original cluster formation event that coincided with the formation of elliptical galaxy and the other two, one from accreted spiral galaxy and other from tidally stripped dwarf galaxies. Hence we may conclude from the above discussion that the intermediate parts of massive elliptical is formed via major merger with the high redshift galaxies in $0.5 \leq z < 0.75$, whose median mass and size are respectively $10^{10.87} M_\odot$ & 2.34 kpc respectively.

In the limit when $< v_a^2 > \ll < v_i^2 >$ or $\varepsilon \ll 1$, the size increases by a factor of four (minor merger). In the present case, the outermost parts of massive ellipticals have sizes

Table 1: Multivariate multisample test for the matching of parent distributions corresponding to data sets 4-8 (at 0.5 percent level of significance)

Sample1	Sample2	p-value	Decision
<i>Damjanov</i> et al.(2011)	<i>Grogin</i> et al.(2011)	0.005	Accepted
, ,	<i>Conscience</i> et al.(2011)	0.007	Accepted
, ,	<i>Nilsson</i> et al.(2011)	0.0447	Accepted
, ,	<i>Mclure</i> et al.(2012)	0.003	Accepted
, ,	<i>Saccaro</i> et al. (2011)	0.096	Accepted
, ,	<i>Papovich</i> et al.(2012)	0.000	Rejected

much larger (median value $\langle R_e \rangle \sim 10.54$ kpc) than innermost part. Also, median mass of this part is of the order of $10^{10.6839} M_\odot$ which is comparable to the combined masses of few dwarf galaxies. So, it might be suspected that the outermost part is primarily composed of stellar components of tidally accreted satellite dwarf galaxies. This is also consistent with our previous works (Chattopadhyay et al. (2009); Chattopadhyay et al. (2013)) in case of NGC 5128. Since data set 3 has no correlations with any subset of high redshift galaxies, we cannot specifically confirm their formation epoch but we can at most say that their formation process is different from the innermost and intermediate part.

Finally we can conclude that formation of nearby massive ellipticals have three parts, inner, intermediate and outermost, whose formation mechanisms are different. The innermost parts are descendants of high ETGs called ‘red nuggets’. The intermediate parts are formed by major mergers in the redshift zone, $0.3 \leq z < 0.75$. The outer envelop might be formed by minor mergers with tidally stripped satellite dwarf galaxies (Mihos et al. (2013); Mondal et al. (2008); Chattopadhyay et al. (2009); Chattopadhyay et al. (2013)). Since, the densities and velocity dispersion values and abundances are not available with the present data sets, so more specific conclusions can be drawn if these data are available for massive ellipticals and satellite dwarfs. But at this moment we can say, that since two different formation scenario are very unlikely for the same galaxy at a particular epoch, so the above study is indicative of a ‘third phase’ of formation of the outermost parts of massive nearby ellipticals rather than a ‘two phase one’ as indicated by previous authors.

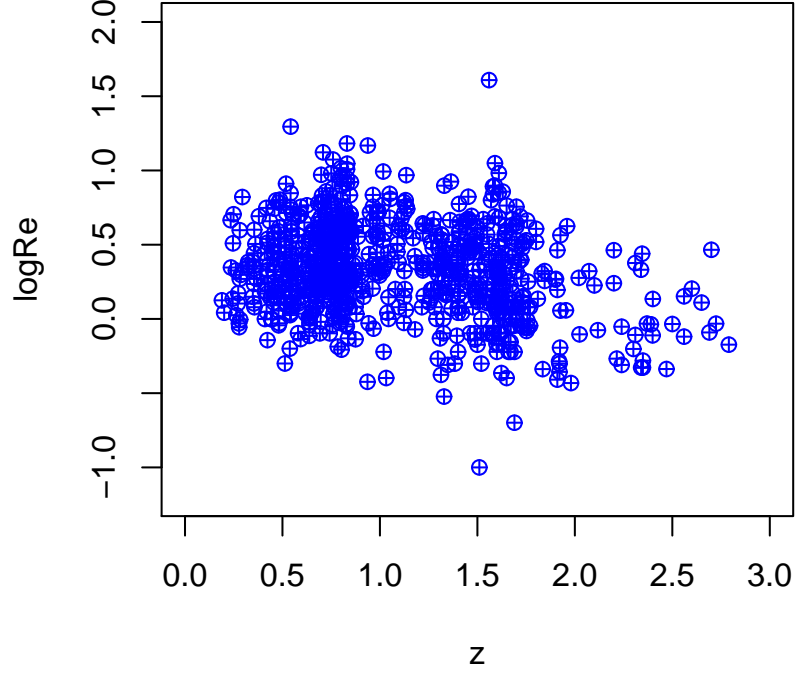


Figure 1: Logarithm of the effective radius versus redshift plot for the entire sample of ETGs in $0.2 \leq z \leq 2.7$

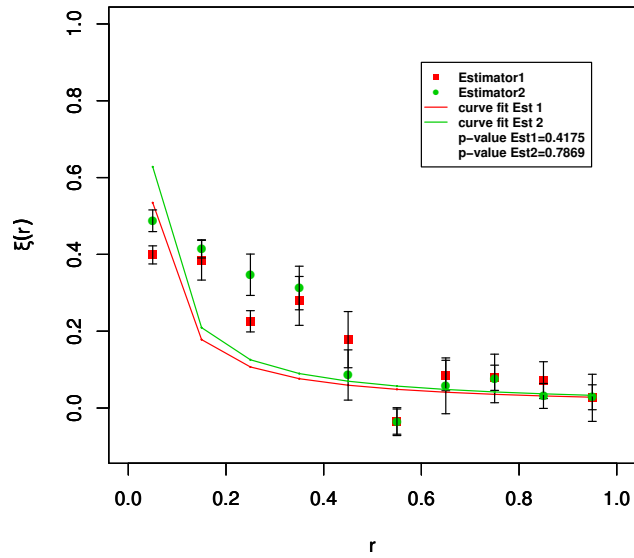


Figure 2: Cross-correlation function $\xi(r)$ versus normalized distance bin r between data sets 1 and 8. The solid lines are power laws for both the estimates as $\xi(r) \propto \frac{1}{r}$

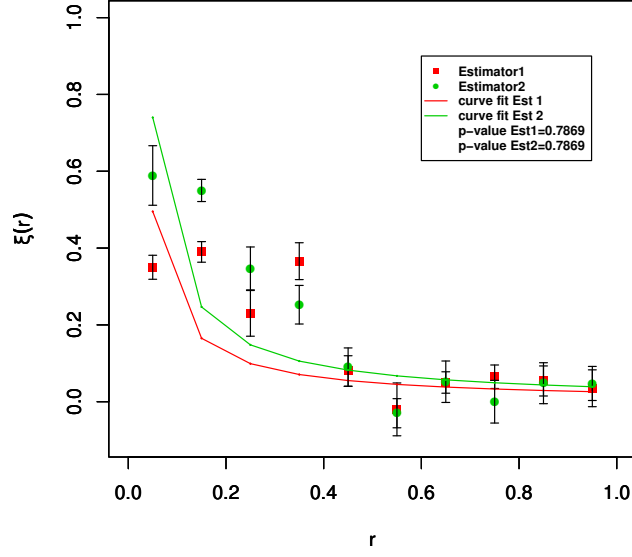


Figure 3: Cross-correlation function $\xi(r)$ versus normalized distance bin r between data sets 2 and 4. The solid lines are power law fits for both the estimates as $\xi(r) \propto \frac{1}{r}$

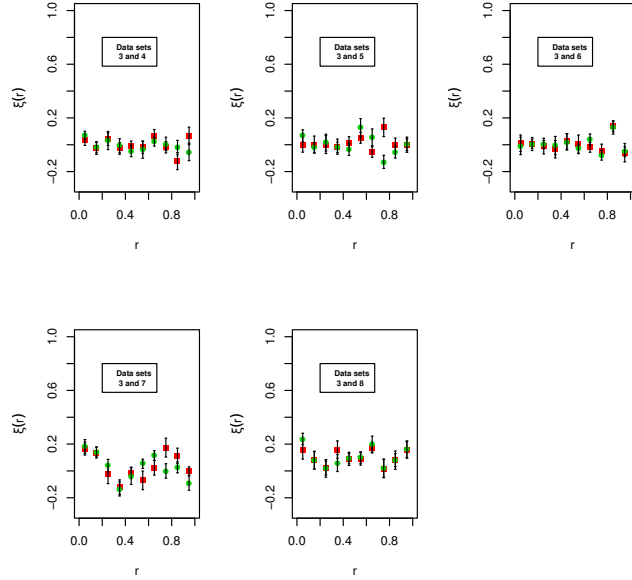


Figure 4: Cross-correlation function $\xi(r)$ versus normalized distance bin r between data set 3 and all redshift bins

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5. Appendix

Multivariate multisample matching test:

For studying the compatibility among data under a multivariate set up, the equality of location measures (mean, median, mode etc.) and dispersion measures (sd, range etc) are of interest. If the joint distribution of the parameters under consideration is multivariate normal then MANOVA test is appropriate otherwise use of a multivariate two sample non-parametric method is a better option. A short description of the multivariate non parametric test used is given below.

Let

$$X_{\alpha}^{(k)} = (X_{1\alpha}^{(k)}, \dots, X_{p\alpha}^{(k)})', \alpha = 1, \dots, n_k, k = 1, \dots, c$$

be a set of independent vector-valued random values, where c is the total number of populations, n_k is the sample size of the k th population and p is the total number of parameters. The cumulative distribution function (c.d.f.) of $X_{\alpha}^{(k)}$ is denoted by $F_k(x)$. The set of admissible hypotheses designates that each $F_k(x)$ belongs to same class of distribution functions Ω . The hypothesis to be tested, say H_0 , specifies that

$$H_0 : F_1(x) = \dots = F_c(x) = F(x), \forall x, \text{ where } F \in \Omega.$$

The alternative to H_0 is the hypothesis that each $F_k(x)$ belongs to Ω but that H_0 does not hold. To avoid the problem of ties, it is assumed that the class Ω is the class of all continuous distribution functions.

Here we pay particular attention to translation-type alternatives. For translation-type alternatives, we let

$$F_k(x) = F(x + \delta_k), \forall k = 1, \dots, c, F \in \Omega,$$

and we are interested in testing (the reversed null hypothesis)

$$H_0^1 : \delta_1 = \dots = \delta_c = 0$$

against the alternative that $\delta_1, \dots, \delta_c$ are not all equal.

We use the "Basic Rank Permutation Principle" given by Puri & Sen (1970). Let us rank the N i -variate observations $X_{i\alpha}^{(k)}$, $\alpha = 1, \dots, n_k$, $k = 1, \dots, c$ in ascending order of magnitude, and

let $R_{i\alpha}^{(k)}$ denote the rank of $X_{i\alpha}^{(k)}$ in this set. The observation vector $X_{\alpha}^{(k)} = (X_{1\alpha}^{(k)}, \dots, X_{p\alpha}^{(k)})'$ then gives rise to the rank vector $R_{\alpha}^{(k)} = (R_{1\alpha}^{(k)}, \dots, R_{p\alpha}^{(k)})'$, $\alpha=1, \dots, n_k$, $k=1, \dots, c$. The N rank vectors corresponding to the N observation vectors, $N = n_1 + n_2 + \dots + n_c$, can be represented by the rank matrix

$$R_N^{p \times N} = \begin{pmatrix} R_{11}^{(1)} & \cdot & \cdot & \cdot & R_{1n_1}^{(1)} & \cdot & \cdot & \cdot & R_{1n_c}^{(c)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ R_{p1}^{(1)} & \cdot & \cdot & \cdot & R_{pn_1}^{(1)} & \cdot & \cdot & \cdot & R_{pn_c}^{(c)} \end{pmatrix}$$

Each row of this matrix is a random permutation of the numbers $1, 2, \dots, N$. Thus, $R_N^{p \times N}$ is a random matrix which can have $(N!)^p$ possible realizations. Two rank matrices of the above form are said to be permutationally equivalent if one can be obtained from the other by a rearrangement of its columns. Thus a matrix R_N is permutationally equivalent to another matrix R_N^* which has the same column vectors as in R_N but they are so arranged that the first row of R_N^* consists of the numbers $1, 2, \dots, N$ in the natural order i.e.

$$R_N^{*p \times N} = \begin{pmatrix} 1 & 2 & \cdot & \cdot & \cdot & N \\ R_{21}^* & \cdot & \cdot & \cdot & \cdot & R_{2N}^* \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ R_{p1}^* & \cdot & \cdot & \cdot & \cdot & R_{pN}^* \end{pmatrix}$$

In order to perform a "Permutation Rank Order Test" we start with a general class of rank scores defined by explicitly known functions of the ranks $1, \dots, N$, viz.,

$$E_{N,\alpha}^{(i)} = \left(\frac{\alpha}{N+1} \right),$$

$$1 \leq \alpha \leq N, i = 1, \dots, p.$$

Now, replacing the ranks $R_{i\alpha}^{(k)}$ in R_N by $E_{N,R_{i\alpha}^{(k)}}^{(i)}$, for all $i = 1, \dots, p$, $\alpha = 1, \dots, n_k$, $k=1, \dots, c$, we get a corresponding $p \times N$ matrix of general scores, which we denote by E_N . Thus,

$$E_N = \begin{pmatrix} E_{N,R_{11}^{(1)}}^{(1)} & \cdot & \cdot & E_{N,R_{1n_1}^{(1)}}^{(1)} & \cdot & \cdot & E_{N,R_{1n_c}^{(c)}}^{(1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ E_{N,R_{p1}^{(1)}}^{(p)} & \cdot & \cdot & E_{N,R_{pn_1}^{(1)}}^{(p)} & \cdot & \cdot & E_{N,R_{pn_c}^{(c)}}^{(p)} \end{pmatrix}$$

We then consider the average rank scores for each i ($=1, \dots, p$) of the c samples, defined by

$$T_{Ni}^{(k)} = \frac{1}{n_k} \sum_{\alpha=1}^{n_k} E_{N,R_{i\alpha}^{(k)}}^{(i)}, k = 1, \dots, c, i = 1, \dots, p.$$

Then, by straightforward computations,

$$v_{ij}(R_N^*) = \frac{1}{N} \sum_{k=1}^c \sum_{\alpha=1}^{n_k} E_{N\alpha,i}^{(k)} E_{N\alpha,j}^{(q)} - \bar{E}_N^{(i)} \bar{E}_N^{(j)}$$

can be obtained, where $E_{N\alpha,i}^{(k)}$ is the value of $E_{N,S}^{(i)}$ associated with the rank $S = R_{i\alpha}^{(k)}$, and

$$\overline{E}_N^{(i)} = \sum_{\alpha=1}^N E_{N,\alpha}^{(i)} / N, i = 1, \dots, p,$$

Now denoting

$$V(R_N^*) = ((v_{ij}(R_N^*)))_{i,j=1,\dots,p},$$

and following the structure of the test asymptotically equivalent to chi-square statistic, we take as our test statistic \mathcal{L}_N ,

$$\mathcal{L}_N = \sum_{k=1}^c n_k [(T_N^{(k)} - \overline{E}_N) V^{-1}(R_N^*) (T_N^{(k)} - \overline{E}_N)']$$

where $V^{-1}(R_N^*) = ((v_{ij}(R_N^*)))^{-1}$, $T_N^{(k)} = (T_{N1}^{(k)}, \dots, T_{Np}^{(k)})$ and $\overline{E}_N = (\overline{E}_N^{(1)}, \dots, \overline{E}_N^{(p)})$.

Let p = number of parameters, c = number of populations,

$N = \sum_{i=1}^c n_i$, n_i is the sample size of the i th sample,

$i=1, \dots, c$, $m_H = c-1$, $m_E = n-c$. The statistic \mathcal{L}_N can be approximated by $m_E c F$,

where F follows F distribution with a, b degrees of freedom.

Here $a = pm_H$, $b = 4 + \frac{a+2}{B-1}$, $c = \frac{a(b-2)}{b(m_E-p-1)}$, where $B = \frac{(m_E+m_H-p-1)(m_E-1)}{(m_E-p-3)(m_E-p)}$.

This approximation was done by McKeon (1974). In order to compute the value of the statistic we have used R - code.